

# Pion electric polarizability from lattice QCD

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**Abstract.** Electromagnetic polarizabilities are important parameters for understanding the interaction between photons and hadrons. For pions these quantities are poorly constrained experimentally since they can only be measured indirectly. New experiments at CERN and Jefferson Lab are planned that will measure the polarizabilities more precisely. Lattice QCD can be used to compute these quantities directly in terms of quark and gluon degrees of freedom, using the background field method. We present results for the electric polarizability for two different quark masses, light enough to connect to chiral perturbation theory. These are currently the lightest quark masses used in polarizability studies.

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## INTRODUCTION

Hadron electromagnetic polarizabilities parametrize the response of the charge and current distribution to an external electromagnetic field. They encode information about the structure of the hadron. More specifically, to the lowest orders in the field strength the interaction between a hadron and an external electromagnetic field is parametrized by the effective hamiltonian:

$$H_{\text{em}} = -\boldsymbol{\mu} \cdot \mathbf{B} - \frac{1}{2}\alpha \mathbf{E}^2 - \frac{1}{2}\beta \mathbf{B}^2 + \dots, \quad (1)$$

where  $\boldsymbol{\mu}$  is the magnetic dipole of the hadron,  $\alpha$  is its electric polarizability, and  $\beta$  is the magnetic one. Here we will focus on the electric polarizability for the pions.

For pions chiral perturbation theory ( $\chi$ PT) predicts that at  $\mathcal{O}(p^4)$ , the leading order contribution for polarizabilities, we have  $\alpha_\pi + \beta_\pi = 0$  [1]. Using this relation and an  $SU(3)$  chiral Lagrangian, pion electric polarizabilities at  $\mathcal{O}(p^4)$  are [2]

$$\begin{aligned} \alpha_{\pi^\pm} &= \frac{4\alpha}{m_\pi f_\pi^2} (L_9^r + L_{10}^r) \approx 2.68(42) \times 10^{-4} \text{ fm}^3, \\ \alpha_{\pi^0} &= -\frac{\alpha}{96\pi^2 m_\pi f_\pi^2} \approx -0.50 \times 10^{-4} \text{ fm}^3. \end{aligned} \quad (2)$$

A review of these calculations and other theoretical inputs relevant for pion polarizability can be found in the theory review included in the DaΦne Physics handbook [3].

Experimentally, for stable hadrons, the polarizabilities are determined from Compton scattering. For pions experimental information comes from Primakoff processes, radiative pion photoproduction and photon-photon fusion. For charged pions the results indicate that  $\alpha_{\pi^\pm} \in (+2, +6) \times 10^{-4} \text{ fm}^3$  and for neutral pions the current results point to  $\alpha_{\pi^0} \in (-1.0, -0.3) \times 10^{-4} \text{ fm}^3$ . In some of these experiments experimental data is used to extract  $\alpha_\pi - \beta_\pi$  and the value of  $\alpha_\pi$  is computed using the  $\chi$ PT prediction  $\alpha_\pi + \beta_\pi = 0$ . Historically, there was some discrepancy between experimental and theoretical predictions for charged pion polarizability, but more recent experiments seem to agree with  $\chi$ PT prediction [4]. New experiments are planned at JLAB and COMPASS that will reduce significantly the uncertainty for these quantities.

Given the rather uncertain experimental and theoretical situation, it is useful to have a calculation for these polarizabilities directly in terms of the quark and gluon degrees of freedom. Lattice QCD can be used to perform such a calculation using the background field method: hadron correlators are computed with and without an external field. The shift due to the external field is used to extract the polarizability. Such calculations are challenging since the effects of polarizability are small and they have to be disentangled from other systematic corrections due to finite volume effects and other lattice artifacts. Additionally, for charged pions, the leading order effect of the external electromagnetic field is to accelerate the hadrons and careful analysis is required to separate the polarizability contribution. A first calculation

of this type [5] was carried out with rather heavy quarks (corresponding to pion masses of 390 MeV) and using rather large background fields. Here we present our results for two lighter quark masses, corresponding to pion masses of 306 MeV and 227 MeV, using much weaker electric fields, well within the region where the effects of the external fields are expected to be dominated by the polarizability contributions. For charged pions, we also introduced an improved method of extracting the polarizabilities that takes into account the finite volume corrections which are important for the lattice sizes used in current simulations.

## CALCULATION DETAILS

In this section we review the methods used to extract the electric polarizability from lattice QCD. We use the background field method [6]. For neutral particles this amounts to computing the hadron mass with and without a background electric field and use Eq. 1 to extract the polarizability from the energy shift. In the continuum the electric field is introduced by coupling it to the covariant derivative

$$D_\mu = \partial_\mu - igG_\mu - iqA_\mu, \quad (3)$$

where  $A_\mu$  is the vector potential corresponding to the background field and  $q$  is the charge of the quark. On the lattice this amounts to a modification of the gauge links by a *em-factor* controlled by the vector potential

$$U'_\mu = e^{-iqaA_\mu} U_\mu, \quad (4)$$

where  $a$  is the lattice spacing. One thing to note is that real electric fields lead to a real *em-factor* on the lattice, since the lattice formulation involves Euclidean time. When the theory is analytical in the strength of the electric field  $\mathcal{E}$  around  $\mathcal{E} = 0$ , then we can use imaginary values for the background electric field which lead to a  $U(1)$  *em-factor*. The only change is that the value of the mass shift is connected to the polarizability using a slightly modified expression. To be more specific, if we use imaginary electric fields, one possible gauge choice is

$$U'_1 = e^{-iaq\mathcal{E}t} U_1 \quad \text{and} \quad U'_\mu = U_\mu \text{ for } \mu \in \{2, 3, 4\}. \quad (5)$$

In this case the polarizability is connected to the phase shift via [7, 8]

$$\Delta m = +\frac{1}{2}\alpha\mathcal{E}^2. \quad (6)$$

We mentioned that we need analyticity in order to be able to use imaginary values for the electric field. In the infinite volume and even on finite boxes with periodic boundary conditions the theory is not analytic around the  $\mathcal{E} = 0$  point due to the Schwinger mechanism. The instability is due to the fact that the cost of creating a charged particle-antiparticle pair can be compensated by aligning the resultant dipole with the electric field. In infinite volume the dipole can be made arbitrarily large by separating the pair and the instability can then be triggered by arbitrarily small external fields. This instability can be removed by limiting the maximal distance in the direction of the field. In our simulations we use Dirichlet boundary conditions in the direction of the field. In the other spatial directions we use periodic boundary conditions. Note that the Dirichlet boundary conditions are only used for the valence quark fields. The gluon fields use periodic boundary conditions in all directions.

One consequence of using Dirichlet boundary conditions is that the hadron cannot be at rest. The lowest momentum is controlled by the distance between the hard walls,  $L$ , and it is of the order  $\pi/L$ . To account correctly for this motion we compute the mass shift using the energy shift measured for moving hadrons. Assuming the continuum dispersion relations,  $E^2 = m^2 + p^2$ , and the fact that the energy shift is very small when compared to the mass of the hadrons we have

$$\Delta m = \frac{E}{m}\Delta E, \quad (7)$$

where  $\Delta E$  is the energy shift,  $E$  is the energy of the moving hadron for zero external field, and  $m$  is the mass of the hadron which is calculated using periodic boundary conditions. This does not account for all possible effects associated with the hadron motion and it is important to carry out an extrapolation to infinite volume. However, only the size in the direction of the field needs to be increased since the finite volume effects in the transverse directions are expected to decay exponentially fast and should be negligible for the lattice sizes used in our calculations.

**TABLE 1.** Parameters for the ensembles used in this study. For each ensemble a number of valence quark masses were used; we indicate here the pseudo-scalar masses as a proxy for the valence quark mass. The values indicated with bold figures correspond to the unitary points, that is the points where the valence and sea quark masses are equal.

| ensemble | size             | spacing     | $N_{\text{cfg}}$ | $m_1$             | $m_2$             | $m_3$      | $m_4$      | $m_5$      |
|----------|------------------|-------------|------------------|-------------------|-------------------|------------|------------|------------|
| EN1      | $24^3 \times 48$ | 0.125(2) fm | 300              | 269(1) MeV        | <b>306(1) MeV</b> | 339(1) MeV | 368(1) MeV | —          |
| EN2      | $24^3 \times 64$ | 0.122(1) fm | 450              | <b>227(2) MeV</b> | 283(1) MeV        | 322(1) MeV | 353(1) MeV | 382(1) MeV |

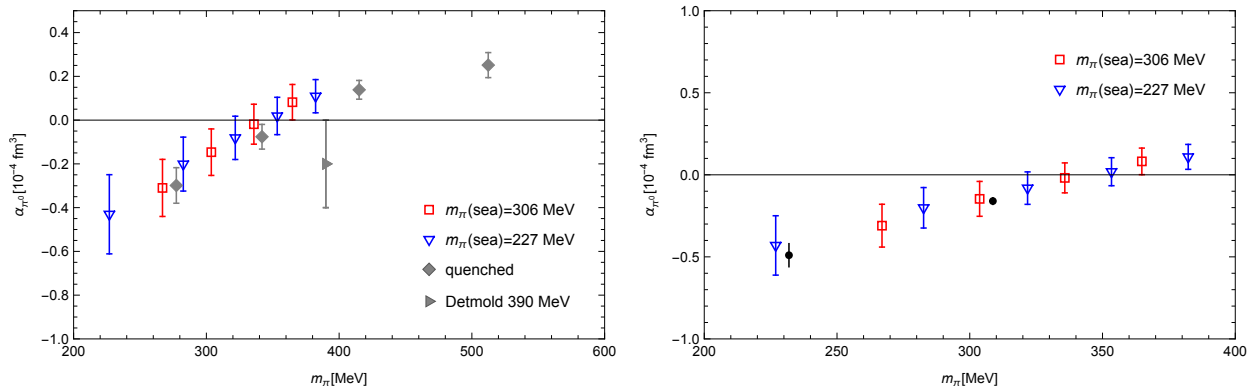
For our calculations we use ensembles with two mass-degenerate sea-quark flavors. The discretization used for both sea quarks and valence quarks is nHYP clover fermions [9]. We use two different sea quark masses, one corresponding to a pion mass of 306 MeV and the other corresponding to 227 MeV. For each ensemble we computed the polarizability for a number of valence quark masses. This allows us to gauge the sensitivity of the polarizability on the sea quark mass and the valence mass separately. The parameters for each ensemble are listed in Table 1. These are the same ensembles we used in a study of neutron electric polarizability where the method described here was tested [10]. Unless otherwise noted these calculations were carried out using neutral sea quarks, with periodic boundary conditions in all directions. We use around 20 point sources for each configurations, and 5 different couplings to the background field for each source, which require thousands of inversions for each configuration. To compute these efficiently we use our own GPU Wilson dslash kernel implementation [11] with an efficient multi-mass inverter [12].

## NEUTRAL PIONS

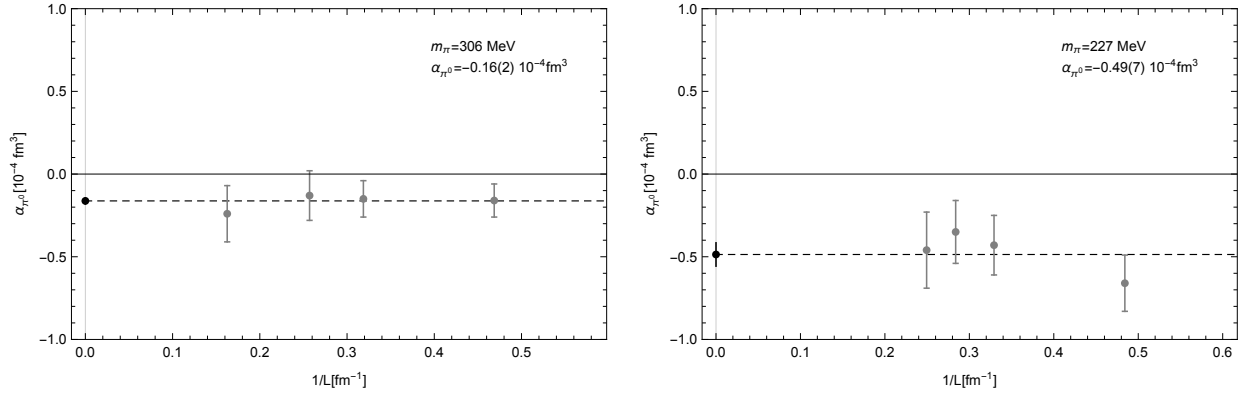
In this section we discuss our results for the electric polarizability of neutral pions. We first note that the correlator for  $\pi^0$  has disconnected contributions even when the sea quarks flavors have the same mass. The disconnected contributions only vanish if the theory is isospin symmetric, which is not the case when the quarks are charged differently. The disconnected contributions are very difficult to compute and are usually not included in lattice calculations since their effect is expected to be small. For electric polarizability the effect of excluding the disconnected contributions is expected to be more significant:  $\chi$ PT predicts that the value of the polarizability will be small and positive [5] and relatively insensitive to the quark mass. In effect, the particle described by the lattice calculations is similar to the pion with the flavor content  $\bar{d}\gamma_5 u$  in a theory where both  $u$  and  $d$  quarks have the same charge.

Our results are presented in Fig. 1. In the left panel we show the results for both EN1 and EN2, together with the results from a previous quenched study. We see that the polarizability is sensitive to the valence quark mass, turning negative for  $m_\pi \approx 350$  MeV. On the other hand, the polarizability seems to be insensitive to the mass of the sea quarks, since the curves for quenched calculation and our results for EN1 and EN2 agree with each other within the error-bars.

These results are a bit different from the expectations from  $\chi$ PT, but they are consistent with results from previous



**FIGURE 1.** Neutral pion electric polarizability as a function of the valence quark mass. Left panel: values extracted on the ensembles EN1 and EN2. The quenched data is taken from [13] and the  $m_\pi = 390$  MeV data point is taken from [5]. Right panel: extrapolations to infinite volume (black points) compared to the finite volume values.



**FIGURE 2.** Neutral pion electric polarizability as a function of the size of the lattice in the direction of the electric field for EN1(left) and EN2(right).

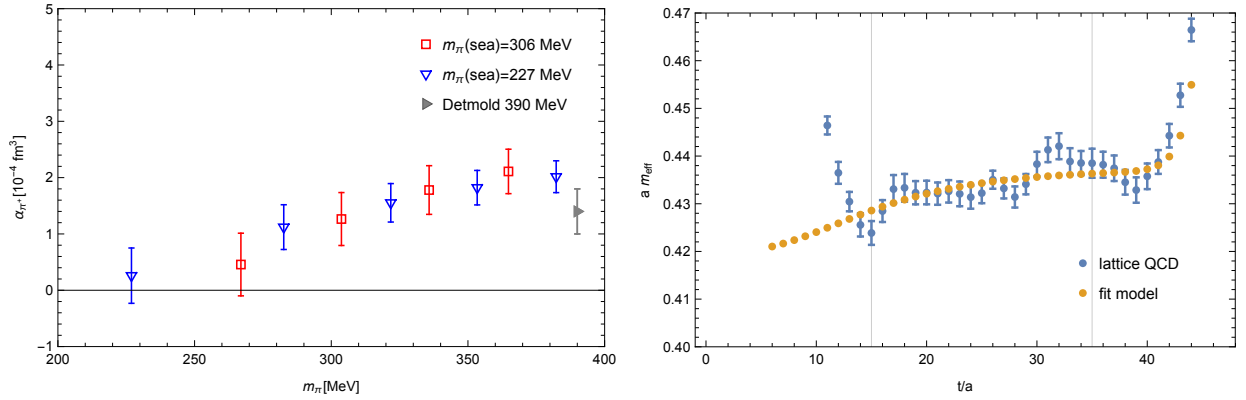
lattice studies [13, 5]. Possible sources for the discrepancy between the lattice results and  $\chi$ PT expectations are the finite volume effects and the neutral sea quarks. To study the finite volume effects, we generated ensembles with the same parameters but with a series of increasing sizes in the direction of the electric field [14]. For pions we found that the polarizability depends very little on the size of the box and we can fit the results with a constant as can be seen in Fig. 2. The results for the infinite volume extrapolation are presented in the right panel of Fig. 1. It is clear that the negative trend remains and that the finite volume effects are not the source of discrepancy. Charging the sea quarks is very challenging. A first study in this direction was carried out recently [15] and the results seem to indicate that the polarizability might turn positive when the sea quarks are charged. However, the error bars are large and no definite conclusion can be derived.

## CHARGED PIONS

Computing the electric polarizability for charged pions presents additional challenges since the hadrons are accelerated by the external field and their correlators are no longer described by a sum of exponential functions. Traditional lattice spectroscopy methods cannot be used to extract the mass of the hadrons. A solution for this problem was proposed in [5]: compute the correlator for a charged particle in a constant electric field and use this functional form to fit the lattice correlator. The range of the fit has to be adjusted so that the lattice correlator is dominated by the lowest mass particle. The functional form for the correlator was derived using Schwinger proper time techniques using imaginary electric fields and Euclidean time.

Our original attempts to fit this functional form to our correlators failed. The cause for this was the finite volume effects. In Fig. 4 we plot the infinite-volume correlator against a free scalar correlator computed in a finite box using lattice methods. Note that we only see an agreement when using large boxes and strong electric fields, much stronger than the one we used in our study. To account for this discrepancy we decided to fit our lattice QCD data to finite-volume scalar propagators computed using numerical methods. This allows us to match the Dirichlet boundary conditions used in our lattice QCD study with relative ease. In the right panel of Fig. 3 we show the effective mass for a lattice QCD correlator together with the fitted result using the scalar corrector. The agreement is very good in the fit window indicated in the figure.

Using this fitting function we perform a correlated fit where both the zero and non-zero field residues are minimized. The mass shift is then converted to polarizability using Eq. 6. The results for our ensembles are shown in the left panel of Fig. 3. We note again that the polarizability decreases with decreasing valence quark mass, but it remains positive throughout the range of the masses used in this study. The results are fairly insensitive to the sea quark mass (as it is the case for neutral pions), but this might be an artifact of the fact that the sea quarks are not charged. The values are smaller than the expectation from  $\chi$ PT and the discrepancy increases as we lower the quark mass towards the physical point.



**FIGURE 3.** Left: electric polarizability for the charged pions as a function of the valence quark mass. The data for  $m_\pi = 390$  MeV is taken from [5]. Right: effective mass for a charged pion correlator together with the scalar particle correlator determined from the fit. The fitting range is indicated by the vertical bars.

## FUTURE DIRECTIONS

Lattice QCD calculations of electric polarizabilities for pions are challenging. The two basic challenges, common to all hadrons, are the fact that the finite volume effects vanish only algebraically in the infinite volume limit and that charging the sea quarks is numerically expensive. Our initial studies of these effects are very promising [14, 15]. To reduce the error-bars, especially as we reduce the quark mass towards the physical point, further improvements are needed. A better understanding of the finite-volume corrections in the presence of Dirichlet boundary conditions, hopefully with a functional form derived from effective field theory, would greatly improve the efficiency of lattice calculations. Perturbative reweighting is effective in taking into account the charge of the sea quarks, and we are currently working on improvements that should allow us to extend this method to larger lattices to compute the infinite volume limit.

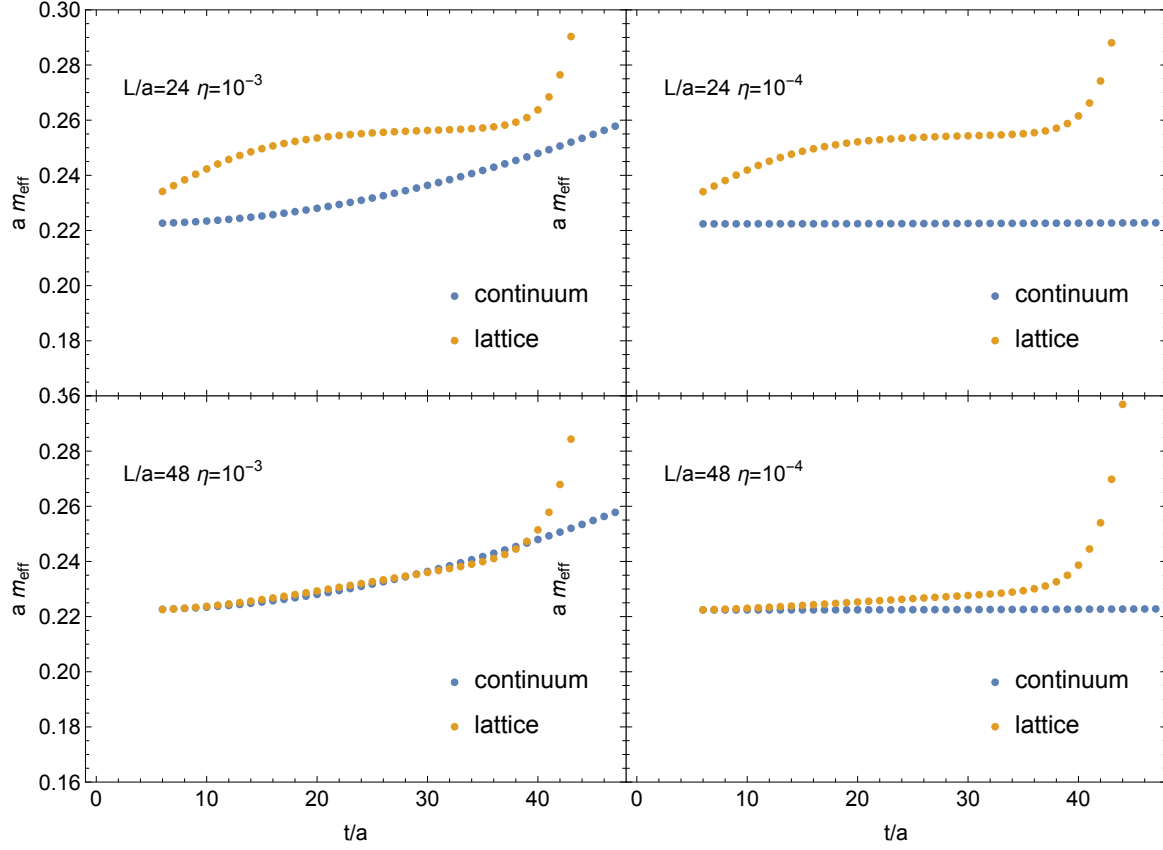
For pions new challenges arise: for neutral pions the inclusion of disconnected diagrams and for charged pions the acceleration due to the external field produces non-standard correlators. The infinite volume correlator cannot be used to fit lattice QCD results unless we use large boxes and strong fields. The method based on the finite-volume scalar correlator produces reasonable results, but we need to test it more thoroughly. We plan to compute the infinite volume limit for the electric polarizability for charged pions and compare it with the experimental results and  $\chi$ PT predictions. We note that for charged baryons a different fitting function is needed. The technique used in this paper can be trivially extended to compute this fitting function.

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**FIGURE 4.** Effective mass for a scalar particle correlator in the presence of a background field. The top panels correspond to a “small” box and the lower ones to a larger box. The left panels correspond to a strong electric field  $\eta \equiv qa^2\mathcal{E} = 10^{-3}$ , whereas the right ones correspond to a weaker field  $\eta = 10^{-4}$ , similar to the one used in our lattice study. The infinite volume result is labeled “continuum”.

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